UMR 1041 INRA – AGROSUP

CESAER

Centre d'Economie et Sociologie
appliquées à l'Agriculture et aux Espaces Ruraux

Are the gains from a groundwater management policy so low?

Elsa Martin

Working Paper

2010/2

UMR 1041 CESAER INRA – AGROSUP
26, Bd Dr Petitjean - BP 87999 - 21079 Dijon cedex

Are the gains from a groundwater management policy so low?

Elsa MARTIN*, CESAER UMR INRA - AgroSup Dijon

March 29, 2010

Abstract

The point of departure of this work is Gisser and Sanchez' (1980) result according to which a groundwater management policy would not generate significant gains with respect to a situation with no control. The theoretical result is obviously checked for a fixed number of agents if the storage capacity of the aquifer is relatively large.

We propose to add an entry component into Rubio and Casino's (2001) adaptation from Gisser and Sanchez' (1980) seminal model in order to make the number of farmers exploiting the resource endogenous. We then show that, at the steady state, the Gisser and Sanchez' result is not true anymore since the rent at the stationary equilibrium is zero, although it is positive if a central planner intervenes.

Keywords: Groundwater management, Entry, Steady-state analysis.
J.E.L. classification numbers: Q10, Q25, Q28.

1. Introduction

Serious depletion of aquifers is a major threat to many freshwater ecosystems all over the world. From a public economics point of view, this phenomenon is due to the inefficiencies of its exploitation and the main question of interest is how to correct them. But what is the size of the gains from a groundwater management policy? It is the issue that we want to address in this paper.

In the seminal work of Gisser and Sanchez (1980) the "competitive" solution is analytically compared with the one of optimal control. The competitive solution is a common property situation with a fixed number of groundwater exploiters. Gisser and Sanchez' theoretical prediction is that if the storage capacity of the aquifer were relatively large, the two systems would be very close. The economic intuition for this is quite obvious since, at the limit, the storage capacity is so large that each agent becomes atomist, having no impact anymore on the groundwater stock when exploiting it.

*Adress: CESAER UMR INRA - Agrosup Dijon 26, Bd du Docteur Petitjean BP 87 999 21 079 Dijon Cedex FRANCE tel: +33(0)3 80 77 26 91 fax: +33(0)3 80 77 25 71 e-mail: elsa.martin@dijon.inra.fr

Gisser and Sanchez's model is based on the assumption that the economic benefits estimated using a consumptive irrigation water demand function and those estimated using an irrigation water demand function based on an application rate are commensurate. However, Kim and Schaible (2000) brought to the fore that they are not. As a consequence, they show that a model like Gisser and Sanchez' one overestimates economic benefits resulting from groundwater irrigation water use, and that the magnitude of the estimation biais is proportional to the rate of irrigation water losses through leaching, runoff and evaporation. This result pleads in favor of a large size of the gains from a groundwater management policy but at the limit, the intuition previously stated is still at work.

Rubio and Casino (2001) confirmed Gisser and Sanchez' prediction when a strategic setting is added into the seminal model. In their framework, agents are able either to pursue path (open-loop) or decision rule (closed-loop or feedback) strategies. In order to model this, they adapted the Gisser and Sanchez' model to a common property context by explicitly introducing a fixed number of homogeneous agents who are playing extraction strategies.

Gisser and Sanchez (1980) explain that contrary to fishing harvesting, when dealing with groundwater, the entry is restricted by land ownership. But what about the exit? Kim et al. (1989) introduced such a possibility by considering the presence of a backstop substitute (rainfed agriculture or desalinized water for instance). They use the technique of multistage optimal control that allows identification of dynamic endogenous adaptations to increasing resource scarcity and backstop technology. Koundouri and Christou (2006) used this framework in order to test the persistence of the Gisser and Sanchez' result with and without the presence of a backstop substitute. Their findings is that it persists in the presence of a backstop substitute and not in its absence.

The main motivation of this work is to also consider entry. Gisser and Sanchez' theoretical result is that there are no gains from groundwater management when the storage capacity of the aquifer is relatively large. But the explanation according to which the entry is restricted by land ownership then becomes false. Indeed, the main way through which the storage capacity of an aquifer can become higher lies in the increase of the covered area. An entry phenomena can hence be at work in the groundwater exploitation problem. From the best of our knowledge no paper on the economics of groundwater is addressing this question. Though, the literature on fisheries has long been concerned with such problems of entry since the seminal paper of Gordon (1954).

Our main claim is that the consideration of an entry phenomena in the groundwater exploitation problem can considerably increase the gains from a groundwater policy. Indeed, in the long run, the rents linked to the resource extraction are completely dissipated although it is not the case anymore under a central planner intervention. We thus show that the Gisser and Sanchez' result does not persist if we consider the possibility of entry into the irrigation race. More precisely, we propose to concentrate on Rubio and Casino's (2001) adaptation from Gisser and Sanchez' (1980) seminal model. They postulate a disaggregate water demand function built on the assumptions that all farmers are identical and that their number is fixed. We then propose to consider that farmers face climatic conditions such that they can either cultivate rainfed crops or irrigated one. If they choose irrigated agriculture, they begin to extract groundwater. We incorporate an opportunity cost into the basic farmers' profit function formulation in order to take into account the outside possibility of rainfed agriculture. Because of this opportunity cost being assumed sunk for each farmer and following the economic theory predictions, the efficient solution will become the one of a sole owner who may be imagined as either a private farmer or a government agency that owns complete rights to the exploitation of the groundwater.

In the next section, we will quickly recall the assumptions made by Rubio and Casino (2001) in order to adapt Gisser and Sanchez's (1980) model to a differential game framework. The reader is referred to Rubio and Casino's paper for more details. Section 3 will be devoted to the entry setting that we add into their basic model. We will then conduct, in section 4, a stationary analysis on the basis of this new setting. Finally, we will propose some concluding remarks and possible extensions.

2. The basic model

The basic model proposed by Gisser and Sanchez (1980) is a simplified representation of the economic, hydrologic and agronomic facts that must be considered relative to the irrigator's choice of water pumping. We are going to describe how Rubio and Casino (2001) adapted it. When the number of farmers N equals 1, both of the models coincide.

2.1. The farmer's revenues

The demand for irrigation water is assumed to be a negatively sloped linear function as follows:

$$W_t = g + kP_t,\ k < 0,\ g > 0$$

where W_t is the amount of groundwater pumped at each time t and P_t the price of water. In order to simplify notations, parameters denoting time are subsequently omitted unless otherwise stated.

Rubio and Casino (2001) furthermore postulate that all farmers are identical. The idea behind this symmetry assumption is to be able to solve the game analytically and to evaluate the effects of strategic behavior on private groundwater pumping. As a first step and in order to be in phase with their basic model, we propose to make the same assumption. The authors then propose to write the aggregate rate of groundwater extraction as $W = Nw_i$ where N is the number of farmers and w_i the pumping rate of the farmer i. With such a specification, the aggregate demand of water will always be the same one, the number of pumpers and their extraction rates being adjusted each one on each other in order to check this. The individual demand functions become as follows:

$$w_i = \frac{1}{N}\left(g + kP\right),\ i = 1, ..., N$$

It is important to mention that when the number of farmers increases, the individual demand for water is reduced. Such a specification is slowing down the over-exploitation of the resource caused by congestion effects, i.e. pumping cost externalities. The farmer's i willingness to pay for groundwater use is:

$$\int_0^{w_i} P(w)\,dw = \frac{N}{2k}w_i^2 - \frac{g}{k}w_i,\ i = 1, ..., N$$

In Gisser and Sanchez's model, the cost of extraction depends on the quantity of water extracted and the depth of the water table. Like most groundwater models, costs vary directly with the pumping rate and inversely with the level of the water table (or, equivalently, the stock of water):

$$C(h, W) = (c_0 + c_1 h)\,W,\ c_1 < 0,\ c_0 > 0$$

where h is the height of the aquifer, i.e. the water table elevation above some arbitrary level that is considered as being the bottom of the aquifer by Rubio and Casino, c_0 the fixed (with respect to the aquifer height) cost linked with the hydrologic cone and c_1 the marginal pumping cost per acre foot of water pumped per foot of lift.

As the unit groundwater pumping costs do not depend on the rate of extraction, Rubio and Casino propose to postulate that the individual farmer's withdrawals costs are as follows:

$$C_i(h, w_i) = \frac{1}{N}(c_0 + c_1 h)\,W = (c_0 + c_1 h)\,w_i$$

It is here implicitly assumed that the well pump capacity constraint is nonbinding and that energy costs are constant along time. Moreover, sunk costs, replacement costs, and capital costs in general are ignored in this seminal formulation.

Finally, the farmer's i net revenues per unit of time are equal to the willingness to pay for groundwater minus the extraction costs of this resource:

$$\frac{N}{2k}w_i^2 - \frac{g}{k}w_i - C_i(h, w_i)$$

2.2. The simplified hydraulic model

The hydraulic model from Gisser and Sanchez is based on classical assumptions such as the "bath-tub" one, which consists in postulating that the aquifer has parallel sides and a flat bottom. The differential equation that describes the water table as a function of time is obtained by equating inflows minus outlets with the impact on the water table:

$$AS\ \dot{h} = R + (\gamma - 1)\,W,\ 0 < \gamma < 1$$

where AS denotes the storage capacity of the aquifer: area, A, time storage coefficient, S, which measures the average saturation of water in the aquifer; γ is the constant return flow coefficient of irrigation water; R denotes the deterministic and constant recharge.

3. The myopic competitive solution with an entry possibility

In order to add an entry component into the seminal model, we assume that each farmer who wants to irrigate his lands located above the aquifer compares the benefits of this choice to outside opportunities (rainfed agriculture), which are represented by an opportunity cost, s, assumed sunk for each farmer.

The net farm rent per unit of time is total revenue minus total cost:

$$\frac{N}{2k}w_i^2 - \frac{g}{k}w_i - C_i\left(h, w_i\right) - s$$

where the number of farmers N can now possibly evolve along time.

3.1. The entry setting

In a dynamic perspective, when the revenues from irrigated agriculture exceed the opportunity cost, farmers enter into the irrigated agriculture race. If we now add the fact that this adjustment is taking time, we have the following law of motion that is governing the entry temporal phenomena:

$$\dot{N} = \eta\left(\frac{N}{2k}w_i^2 - \frac{g}{k}w_i - C_i\left(h, w_i\right) - s\right), \; N(0) = N_0 > 1 \tag{3.1}$$

where $\eta > 0$ is an adjustment parameter and i denotes the last farmer who is entering into the irrigated agriculture race. This equation correspond to the seminal model of dynamic entry to a fishery, developed by Smith (1968), which is still used nowadays in the literature on fisheries: see for instance Sanchirico and Wilen (1999) who proposed to add a spatial component in the basic model. But this specification suffers from a vagueness since the literature on fisheries measures the fishing effort as the number of boats that must be an integer variable. Some authors proposed to partially solve this issue by reasoning on a continuous variable like the number of fishing days. The same problem holds in our framework with the number of farmers. We could also have preferred to reason on a number of irrigation days but such a specification would have created another problem: agents' rationality would thus have been ignored. Indeed, with such an interpretation of N, the dynamic in equation 3.1 would have suffered from not being based on an optimization process but on an evolutionary one. It is in order to stay in phase with Rubio and Casino framework that we chose to keep the number interpretation. We hope that the reader will not suffer from this imprecision.

This entry equation is defined in such a way that, in the long run, the number of agents exploiting the resource is characterized by the complete dissipation of the rents. Even if this setting is currently referred as a "tragedy of commons" one, it does not lead inevitably to the disappearance of the natural resource. This is specially true in our model since the number of farmers is entering into their individual water demand function in such a way that the global demand is always the same one, hence having the same impact on the stock whatever the number of agents is. This formulation also induces the presence of intramarginal rents since, in the absence of water markets, farmers collect the consumer surplus, which is not the case in the fishery problem where marginal benefit is equal to the price of the fish. In our framework, each additional farmer will hence also reduces the consumer surplus for other farmers.

3.2. The stationary equilibrium

Contrary to Rubio and Casino, we chose to concentrate on the solution of myopic competitive agents. In such a setting, each firm is a too small part of the whole to give serious considerations to how its pumping decision affects future water supplies. Such agents are not able to consider the intertemporal effect of their current choice. So they maximize their current rents without taking into account the impact of their pumping decision on the groundwater stock. The aquifer height is then determined thanks to the aquifer hydraulic law of motion and the number of agents thanks to the entry dynamic previously described.[1]

Proposition 3.1. *The myopic (denoted m) competitive stationary equilibrium (denoted e) is characterized as:*

$$\begin{aligned} \tilde{N}_e^m &= \left[\frac{-R^2}{2sk\left(\gamma-1\right)^2}\right], \; \tilde{w}_e^m = \frac{2sk\left(\gamma-1\right)}{R} \\ \tilde{h}_e^m &= \frac{-R}{kc_1\left(\gamma-1\right)} - \frac{g}{kc_1} - \frac{c_0}{c_1} \end{aligned}$$

where $\sim$ denotes our entry setting, i.e. with the number of farmers being endogenous and [x] the integer part of x.

Figures A.2 and A.1 (see appendix A) summarizes the steady states analytical expressions when $N = 1$ (Gisser and Sanchez, 1980), when N is fixed (Rubio and Casino, 2001), and when N is endogenous (our modified framework with entry possibility). As it can easily be checked, the aquifer height at the steady state with an entry component is the same one as in the Gisser and Sanchez' competitive case since Rubio and Casino's introduction of the number of farmers does not affect the aggregate groundwater demand: when the number increases, the individual demand for the resource is reduced but the aggregate one remains the same. So an increase in the number of farmers (with respect to one) does not affect the aquifer height.

Proposition 3.2. *The number of farmers at the myopic stationary equilibrium is:*

(i) increasing with the natural recharge, R, and with the percolation coefficient, γ, since more groundwater is then available, and also with the elasticity of the water demand to the resource price, k, since more intramarginal rents are hence generated;

(ii) decreasing with the opportunity cost, s, since it measures the cost of entry into the irrigated agriculture race.

[1] It is obvious to check that this equilibirum is always symmetrical since nothing is differentiating each agent private calculus.

4. The gains from a groundwater management policy

In order to measure the gains from a groundwater management policy when the number of farmers is endogenous, we are now going to compare the entry solution to the socially optimal one. Since the entry model is mainly a stationary one, we will focus on steady states comparisons.

In the socially optimal solution, the entry hypothesis does not hold since a central planner with a perfect foresight is assumed to control both the volume of groundwater pumped but also the number of farmers having an access to the aquifer that is assumed to be adjusted instantaneously. Thus, equation 3.1 does not hold anymore here. The symmetry of this solution is obvious since a central planner maximizing the rents accruing to homogenous farmers has never an incentive to discriminate between them since they are all identical. This planner's objective is to maximize the future value of social rent stream that is given by the sum of the individual ones, taking into account the impact of these decisions on the hydraulic system[2]:

$$\begin{aligned} & \max_{w,\, N} \int_0^\infty N \left[\frac{N}{2k} w^2 - \frac{g}{k} w - (c_0 + c_1 h)\, w - s \right] e^{-rt} dt \\ \text{s.t. } \dot{h} &= \frac{1}{AS} \left[R + (\gamma - 1)\, N w \right],\ h(0) = h_0 > 0 \\ N &\geq 1 \end{aligned}$$

We need to constrain the number of farmers because of the opportunity cost. Indeed, without this constraint, a number of agents lower than one (but different from zero) could be socially optimal.

Proposition 4.1. *The socially optimal stationary solution (denoted *) can be analytically derived as:*

$$\begin{aligned} \tilde{N}_e^* &= 1,\ \tilde{w}_e^* = -\frac{R}{\gamma - 1} \\ \tilde{h}_e^* &= \frac{-R}{kc_1 (\gamma - 1)} - \frac{g}{kc_1} - \frac{c_0}{c_1} + \frac{R}{rAS} \\ \tilde{\lambda}_e^* &= \frac{c_1 R}{r (\gamma - 1)},\ \tilde{\mu}_e^* = s \end{aligned}$$

where λ denotes the shadow price of the aquifer height and μ the Lagrange multiplier associated to the constraint on the number of farmers.

This optimum corresponds to Gisser and Sanchez' one (see appendix A). Indeed, because of the opportunity cost, s, being sunk, the monopoly solution is the more efficient one. The aquifer

[2] As previously, the non-negativity constraints on the control variables are implicitly assumed and $h \geq 0$ is not imposed as a state constraint but as a terminal condition for simplicity: $\lim_{t \to \infty} h(t) \geq 0$.

height at the steady state is also the same one as the Rubio and Casino's efficient one that, as stressed by the authors, does not depend on the number of agents.

We now turn to the comparison of these steady state values with the one obtained in the myopic case in order to check if the result of Gisser and Sanchez still holds. Since $\tilde{N}_e^m$ can not be lower than one and the aggregate demand does not change with the number of farmers (remind that $W = Nw_i$), the individual volume of groundwater pumped at the myopic competitive stationary equilibrium is always lower or equal to the one of the socially optimal solution. Concerning the aquifer heights at the two steady states, we obtain the same difference as in the literature since these values do not depend on the number of agents:

$$\tilde{h}_e^* - \tilde{h}_e^m = \frac{R}{rAS} > 0$$

So our results differ from Rubio and Casino's one simply by the fact that the number of farmers and the individual amounts of groundwater pumped are different. But this is of major interest since it has an impact on the individual rents; we even know from the entry setting that the rents are completely dissipated at the steady state. This leads us to the main point of this work that is obvious to demonstrate.

Proposition 4.2. *Making the number of agents endogenous in Rubio and Casino's model adapted from Gisser and Sanchez' one leads to the same conclusion: when the storage capacity of the aquifer studied is relatively large, the aquifer height at the steady state tends to be the same one in the regime of private extraction and in the socially optimal one. But in such a "tragedy of the commons" setting, private extraction leads to the complete dissipation of the farmers' rent in the long run that destroys the result of Gisser and Sanchez since the myopic competitive rents at the steady state then equal zero and not the one characterizing the socially optimal solution.*

5. Conclusion

The Gisser and Sanchez' effect states that the numerical magnitude of benefits of optimally managing groundwater is insignificant. Our main claim is that it cannot be true when considering that entry is possible since, in such a setting, at the steady state, the rents from irrigated agriculture equal zero, which is not the case within the framework of a central planner intervention. This point considerably differs from the literature on the economics of groundwater that is deliberatively omitting entry phenomena. The main explanation given by authors is that the access to an aquifer is limited by land ownership. But since each farmer can become very small in a competitive setting, the economics concept of entry can be at work even if the area giving an access to the resource is limited.

In order to show this, we chose to follow the framework proposed by Rubio and Casino (2001) that is a priory internalizing some groundwater extraction externalities by constraining the aggregate water demand to be the same one whatever the number of agents is. But even if some of these congestion external effects then disappear, some others are still remaining. And, in a setting in which the number of agents is exogenously fixed, this specificity helps to confirm the Gisser and Sanchez' effect since the externalities do not hold anymore when the aquifer size becomes infinite (agents become a too small part of the whole). It is because of this a priory internalization that our entry setting leads to the complete dissipation of the rent without leading to the disappearance of the natural resource exploited.

The main implication in terms of policy intervention is that when there is entry to the irrigated agriculture with groundwater, there is strong needs to manage this resource. Our model does not tell anything about how to intervene. It simply shows that the fact that the number of agents exploiting the resource is endogenous has an economic impact on their rents that can be very strong (rents at the steady state equal to zero) if no central planner intervenes.

But our model contains limits. First, a complete dynamic analysis of entry would consist in comparing the net present value of future rent stream in the different cases studied. In order to compute this one, it is necessary to fully characterize the paths leading to the steady states studied in this work. The difficulty comes from the entry dynamic equation that introduces nonlinearity into our problem. Then, contrary to Rubio and Casino (2001), we mainly concentrated our argument on the competitive assumption. But it could perhaps have been more concrete and also interesting to consider that the strategic behavior of agents depends on the number they are. We would hence have some regimes corresponding to different types of behavioral assumptions, myopic one prevailing when the number is very high and feedback one when farmers are less numerous.

References

[1] Gisser M., D. Sanchez (1980) "Competition versus optimal control in groundwater pumping", *Water Resources Research* 16(4), 638-642

[2] Gordon H.S. (1954) "The Economic Theory of a Common-Property Resource: The Fishery", *The Journal of Political Economy*, Vol. 62, No. 2, pp. 124-142

[3] Kim C., M. Moore, J. Hanchar, M. Nieswiadomy (1989) "A dynamic model of adaptation to resource depletion: theory and an application to groundwater mining", *Journal of Environmental Economics and Management* 17, 66-82

[4] Kim C.S., G.D. Schaible (2000) "Economic Benefits Resulting From Irrigation Water Use: Theory and an Application to Groundwater Use", *Environmental and Resource Economics* 17, 73–87

[5] Koundouri P., C. Christou (2006) "Dynamic adaptation to resource scarcity and backstop availability: theory and application to groundwater", *The Australian journal of agricultural and resource economics* 50, 227-245

[6] Rubio S., Casino B. (2001) "Competitive versus efficient extraction of a common property resource: The groundwater case", *Journal of Economics Dynamics and Control* 25, 1117-1137

[7] Sanchirico J., Wilen J. (1999) "Bioeconomics of Spatial Exploitation in a Patchy Environment", *Journal of Environmental Economics and Management* 37, 129-150

[8] Smith V.L. (1968) "Economics of production from natural resources", *The American Economic Review* 58(3), 409-431

APPENDIX

A. Synopsis of results

		Aquifer height **h**
Gisser and Sanchez (1980)	**Competitive**	$-\frac{R}{kc_1(\gamma-1)}-\frac{1}{c_1}\left(\frac{g}{k}+c_0\right)$
	Efficient	$-\frac{R}{kc_1(\gamma-1)}+\frac{R}{rAS}-\frac{1}{c_1}\left(\frac{g}{k}+c_0\right)$
Rubio and Casino (2001)	**Myopic**	$-\frac{R}{kc_1(\gamma-1)}-\frac{1}{c_1}\left(\frac{g}{k}+c_0\right)$
	Open-loop	$-\frac{R}{kc_1(\gamma-1)}+\frac{R}{rASN}-\frac{1}{c_1}\left(\frac{g}{k}+c_0\right)$
	Feedback	$-\frac{R}{kc_1(\gamma-1)}-\frac{1}{c_1}\left(\frac{g}{k}+c_0\right)+\frac{R}{NAS\left\{r-\left[k(\gamma-1)(N-1)/NAS\right]\left[c_1-\alpha(\gamma-1)/AS\right]\right\}}$
	Efficient	$-\frac{R}{kc_1(\gamma-1)}+\frac{R}{rAS}-\frac{1}{c_1}\left(\frac{g}{k}+c_0\right)$
With entry	**Myopic competitive**	$-\frac{R}{kc_1(\gamma-1)}-\frac{1}{c_1}\left(\frac{g}{k}+c_0\right)$
	Efficient	$-\frac{R}{kc_1(\gamma-1)}+\frac{R}{rAS}-\frac{1}{c_1}\left(\frac{g}{k}+c_0\right)$

Figure A.1: Aquifer height steady state in the different settings

		Number of farmers N	Private groundwater withdrawals, w	Shadow price of the stock λ
Gisser and Sanchez (1980)	Competitive	1	$-\frac{R}{(\gamma-1)}$	0
	Efficient	1	$-\frac{R}{(\gamma-1)}$	$\frac{c_1 R}{r(\gamma-1)}$
Rubio and Casino (2001)	Myopic	N	$-\frac{R}{(\gamma-1)N}$	0
	Open-loop	N	$-\frac{R}{(\gamma-1)N}$	$\frac{c_1 R}{r(\gamma-1)N}$
	Feedback	N	$-\frac{R}{(\gamma-1)N}$	$\frac{R}{N(\gamma-1)\left\{r-\left[k(\gamma-1)(N-1)/NAS\right]\left[c_1-\alpha(\gamma-1)/AS\right]\right\}}$
	Efficient	N	$-\frac{R}{(\gamma-1)N}$	$\frac{c_1 R}{r(\gamma-1)}$
With entry	Myopic competitive	$\left[\frac{-R^2}{2sk(\gamma-1)^2}\right]$	$\frac{2sk(\gamma-1)}{R}$	0
	Efficient	1	$-\frac{R}{(\gamma-1)}$	$\frac{c_1 R}{r(\gamma-1)}$

Figure A.2: Other steady states

B. Proof of proposition 3.1

In a myopic setting, the first order necessary condition for an interior solution is:

$$\frac{N}{k}w - \frac{g}{k} - (c_0 + c_1 h) = 0$$

Furthermore, since at the steady state $\dot{h} = \dot{N} = 0$, the following system yields:

$$\left\{ \begin{array}{c} \frac{1}{AS}\left[R + (\gamma-1)\tilde{N}_e^m \tilde{w}_e^m\right] = 0 \\ \frac{\tilde{N}_e^m}{2k}\tilde{w}_e^{m2} - \frac{g}{k}\tilde{w}_e^m - \left(c_0 + c_1 \tilde{h}_e^m\right)\tilde{w}_e^m - s = 0 \end{array} \right\}$$

Thus, if we express $\tilde{w}_e^m$ in the first order equation, we obtain:

$$\tilde{w}_e^m = \frac{g}{\tilde{N}_e^m} + \frac{k\left(c_0 + c_1 \tilde{h}_e^m\right)}{\tilde{N}_e^m}$$

We then directly deduce the aquifer height at the stationary equilibrium from $\dot{h} = 0$:

$$\frac{(\gamma-1)kc_1\tilde{h}_e^m}{AS} = -\frac{R}{AS} - \frac{(\gamma-1)g}{AS} - \frac{(\gamma-1)kc_0}{AS}$$

$$\Leftrightarrow \tilde{h}_e^m = -\frac{R}{(\gamma-1)kc_1} - \frac{g}{kc_1} - \frac{c_0}{c_1}$$

and the number of farmers from $\dot{N} = 0$:

$$\left[\frac{g}{\tilde{N}_e^m} + \frac{k}{\tilde{N}_e^m}\left(-\frac{R}{(\gamma-1)k} - \frac{g}{k}\right)\right]\left[\frac{g}{2k} + \left(\frac{1}{2} - 1\right)\left(-\frac{R}{(\gamma-1)k} - \frac{g}{k}\right) - \frac{g}{k}\right] - s = 0$$

$$\Leftrightarrow \frac{-R^2}{2(\gamma-1)^2 k\tilde{N}_e^m} - s = 0$$

$$\Leftrightarrow \tilde{N}_e^m = \frac{-R^2}{2\left(\gamma-1\right)^2 ks}$$

Finally,

$$\tilde{w}_e^m = \frac{-R/\left(\gamma-1\right)}{-R^2/2\left(\gamma-1\right)^2 ks} = \frac{2\left(\gamma-1\right)ks}{R}$$

C. Proof of proposition 3.2

These results directly come from some basic static comparative on the number of agents at the myopic competitive stationary equilibrium:

(i)

$$\begin{aligned}\frac{\partial \tilde{N}_e^m}{\partial R} &= \frac{-R}{sk\left(\gamma-1\right)^2} > 0\\ \frac{\partial \tilde{N}_e^m}{\partial \gamma} &= \frac{R^2}{sk\left(\gamma-1\right)^3} > 0\\ \frac{\partial \tilde{N}_e^m}{\partial k} &= \frac{R^2}{2sk^2\left(\gamma-1\right)^2} > 0\end{aligned}$$

(ii)

$$\frac{\partial \tilde{N}_e^m}{\partial s} = \frac{R^2}{2s^2k\left(\gamma-1\right)^2} < 0$$

D. Proof of proposition 4.1

In order to solve the social planner problem, we propose to use the maximum principle. We hence define the current value Hamiltonian as:

$$\tilde{H}^*(h,w,N,\lambda) = N\left[\frac{N}{2k}w^2 - \frac{g}{k}w - \left(c_0+c_1h\right)w - s\right] + \frac{\lambda}{AS}\left[R + \left(\gamma-1\right)Nw\right]$$

and the Lagrangian as:

$$L(h,w,N,\lambda,\mu) = \tilde{H}^*(h,w,N,\lambda) + \mu\left(N-1\right)$$

The necessary conditions yield:

$$\tilde{N}^*\left[\frac{\tilde{N}^*}{k}\tilde{w}^* - \frac{g}{k} - \left(c_0 + c_1\tilde{h}^*\right) + \frac{\tilde{\lambda}^*\left(\gamma-1\right)}{AS}\right] = 0 \tag{D.1}$$

$$\frac{\tilde{N}^*}{k}\tilde{w}^{*2} - \frac{g}{k}\tilde{w}^* - \left(c_0 + c_1\tilde{h}^*\right)\tilde{w}^* - s + \tilde{\mu}^* + \frac{\tilde{\lambda}^*\left(\gamma-1\right)\tilde{w}^*}{AS} = 0 \tag{D.2}$$

$$\dot{h} = \frac{1}{AS}\left[R + \left(\gamma-1\right)\tilde{N}^*\tilde{w}^*\right] \tag{D.3}$$

$$\dot{\lambda} = r\tilde{\lambda}^* + c1\tilde{w}^*\tilde{N}^* \tag{D.4}$$

along with the transversality conditions:

$$\lim_{t\to+\infty} e^{-rt}\tilde{\lambda}^* \geq 0,\ \lim_{t\to+\infty} e^{-rt}\tilde{\lambda}^*\tilde{h}^* = 0,\ i = 1,...,N$$

and the complementary slackness one:

$$\tilde{\mu}^*(\tilde{N}^*-1) = 0,\ \tilde{\mu}^* \geq 0,\ \tilde{N}^* - 1 \geq 0$$

At the steady state, since $\dot{h} = \dot{\lambda} = 0$ and $\tilde{N}_e^* - 1 \geq 0$,

$$(D.1) \Leftrightarrow \frac{\tilde{N}_e^*}{k}\tilde{w}_e^* - \frac{g}{k} - \left(c_0 + c_1\tilde{h}_e^*\right) + \frac{\tilde{\lambda}_e^*\left(\gamma-1\right)}{AS} = 0 \tag{D.5}$$

$$\begin{aligned}&\Rightarrow \quad \tilde{\mu}^* = s > 0 \text{ in (D.2)}\\ &\Leftrightarrow \quad \tilde{N}_e^* = 1\end{aligned}$$

$$\Rightarrow \left\{ \begin{array}{c} \tilde{w}_e^* = \frac{-R}{\gamma - 1} \text{ in (D.3)} \\ \tilde{\lambda}_e^* = \frac{c_1 R}{r(\gamma - 1)} \text{ in (D.4)} \\ \tilde{h}_e^* = \frac{-R}{c_1 k(\gamma - 1)} - \frac{g}{c_1 k} - \frac{c_0}{c_1} + \frac{R}{rAS} \text{ in (D.5)} \end{array} \right\}$$